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Square Root Singularity in Boundary Reflection Matrix

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Abstract

Two-particle scattering amplitudes for integrable relativistic quantum field theory in 1+1 dimensions can normally have at most singularities of poles and zeros along the imaginary axis in the complex rapidity plane. It has been supposed that single particle amplitudes of the exact boundary reflection matrix exhibit the same structure. In this paper, single particle amplitudes of the exact boundary reflection matrix corresponding to the Neumann boundary condition for affine Toda field theory associated with twisted affine algebras $a_{2n}^{(2)}$ are conjectured, based on one-loop result, as having a new kind of square root singularity.

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1. Introduction

Two-particle scattering amplitudes for massive integrable relativistic quantum field theory in 1+1 dimensions can have ordinary square root threshold singularity in terms of Mandelstam variables[1]. However, a reparametrisation of the energy-momentum of on-shell states in terms of rapidity parameter unfolds the square root threshold singularity so that the scattering amplitudes can have at most zeros and poles along the imaginary axis in the complex rapidity plane, provided they do not exhibit any other sort of branch cuts as is usually supposed[2]. In this case, odd-order poles in the physical strip are to be interpreted as a signal of the existence of virtual bound states, which should be among the initial spectrum of asymptotic states by bootstrap principle. Various aspects of the singularity structure of the proposed exact S -matrices for affine Toda field theory(ATFT) have been extensively studied in terms of the so-called Landau singularity of Feynman diagrams[3, 4, 5, 6].

By the way, the models defined on a space with a boundary, let say on a half line, naturally lead one to consider the boundary reflection matrix in order to describe the reflection process of particles against the boundary. Indeed, since the boundary bootstrap equation[7] and the boundary crossing-unitarity relation[8] were introduced, a variety of solutions of the boundary Yang-Baxter equation for the boundary reflection matrix which was first introduced in [9] has been constructed[7, 8, 10, 11]. In this algebraic approach, S -matrices are used as a part of input data and proper interpretations in the framework of Lagrangian quantum field theory was not given[†].

Classical boundary reflection matrices corresponding to the various choices of the integrable boundary condition[‡] have been constructed by linearising the equation of motion around a background solution in [14, 15], where some conjectures on the corresponding exact boundary reflection matrices have been also made. A study on the boundary reflection matrix in quantum field theory has been initiated in the framework of the Feynman's perturbation theory in [22] and single particle reflection amplitudes for ATFT with the Neumann boundary condition were constructed in [23, 24]. Quite recently, a geometric expression of the boundary reflection matrices in terms of root systems for simply-laced ATFT was obtained in [25].

Single particle amplitudes of the exact boundary reflection matrix have been usually

[†]There are some works which aim to relate physical parameters in the boundary potential to formal parameters arising from solutions of the algebraic equations; for the sine-Gordon theory at a generic point in semi-classical analysis[12] and at the free fermion point[13] where one may use the method[8] of mode expansion for the field as an operator.

[‡]For studies on the possible integrable boundary potentials, see [8, 14, 15, 16, 17, 18, 19, 20, 21].

supposed to exhibit the same analytic structure as two-particle scattering amplitudes in the absence of a new idea. In the mean time, there appeared a first example of single particle amplitudes of the exact boundary reflection matrix having a new kind of square root singularity in the case of $a_2^{(2)}$ theory[22]. In the present paper, for ATFT associated with twisted affine algebras $a_{2n}^{(2)}$ for any n , single particle amplitudes of the exact boundary reflection matrix corresponding to the Neumann boundary condition are constructed and checked, based on one-loop result, as having square root singularities.

The plan of this paper is as follows. Section 2 contains partial results of the single particle amplitudes at one-loop order for $a_{2n}^{(2)}$ ATFT defined on a half line with the Neumann boundary condition. In section 3, a complete set of the exact boundary reflection matrix having square root singularities is presented based on the reduction idea[26, 27, 28] and checked against the one-loop result. Finally, conclusions are made in section 4.

2. Perturbative Boundary Reflection Matrix

The action for ATFT[29] associated with a Lie algebra g with the rank r defined on a half line ($-\infty < x \leq 0$) is given by

$$S(\Phi) = \int_{-\infty}^0 dx \int_{-\infty}^{\infty} dt \left(\frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{m^2}{\beta^2} \sum_{i=0}^r n_i e^{\beta \alpha_i \cdot \Phi} \right) \quad (2.1)$$

where $\alpha_0 = -\sum_{i=1}^r n_i \alpha_i$, and $n_0 = 1$. The field ϕ^a ($a=1, \dots, r$) is a -th component of the scalar field Φ , and α_i ($i=1, \dots, r$) are simple roots of the algebra g . The m sets the mass scale and the integers n_i are the so-called Kac labels which are defined for each Lie algebra.

To extract the boundary reflection amplitudes in the framework of the Feynman's perturbation theory, two-point Green's functions are considered in the coordinate space rather than in the momentum space. At one-loop order, there are three types of the relevant Feynman diagrams.

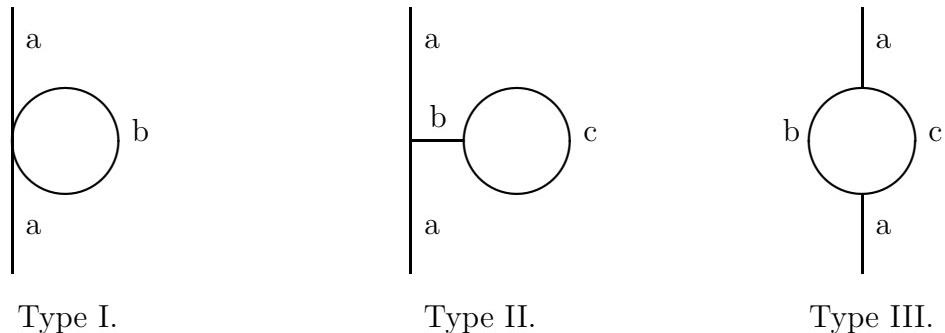


Figure 1. Diagrams for the one-loop two-point function.

After the infinite as well as finite mass renormalization, the remaining terms in the two-point Green's function at one-loop order can be written in the following form in the asymptotic region up to exponentially damped term as x, x' tend to $-\infty$ away from the boundary[22]:

$$\int \frac{dw}{2\pi} e^{-iw(t'-t)} \frac{1}{2\bar{k}} \left(e^{i\bar{k}|x'-x|} + K_a(w) e^{-i\bar{k}(x'+x)} \right), \quad \bar{k} = \sqrt{w^2 - m_a^2} \quad (2.2)$$

Two particle amplitudes of the elastic boundary reflection matrix are defined as the coefficients K_a of the reflection term and $K_a(\theta)$ is obtained using $w=m_a ch\theta$. Each contribution to $K_a(\theta)$ from three types of the diagrams depicted in figure 1 are listed below[22, 23]:

$$K_a^{(I)}(\theta) = \frac{1}{4m_a sh\theta} \left(\frac{1}{2\sqrt{m_a^2 sh^2\theta + m_b^2}} + \frac{1}{2m_b} \right) C_1 S_1 \quad (2.3)$$

$$K_a^{(II)}(\theta) = \frac{1}{4m_a sh\theta} \left(\frac{-i}{(4m_a^2 sh^2\theta + m_b^2)2\sqrt{m_a^2 sh^2\theta + m_c^2}} + \frac{-i}{2m_b^2 m_c} \right) C_2 S_2 \quad (2.4)$$

$$K_a^{(III)}(\theta) = \frac{i}{4m_a sh\theta} C_3 S_3 \quad (2.5)$$

$$\begin{aligned} & \left(\frac{\cos\theta_{ab}^c}{4m_a m_b^2 (ch^2\theta - \cos^2\theta_{ab}^c)} - \frac{m_a ch^2\theta + m_b \cos\theta_{ab}^c}{2m_a m_b^2 2\sqrt{m_a^2 sh^2\theta + m_c^2} (ch^2\theta - \cos^2\theta_{ab}^c)} \right. \\ & \left. + \frac{\cos\theta_{ac}^b}{4m_a m_c^2 (ch^2\theta - \cos^2\theta_{ac}^b)} - \frac{m_a ch^2\theta + m_c \cos\theta_{ac}^b}{2m_a m_c^2 2\sqrt{m_a^2 sh^2\theta + m_b^2} (ch^2\theta - \cos^2\theta_{ac}^b)} \right) \end{aligned}$$

where θ_{ab}^c is the usual fusing angle defined by $m_c^2 = m_a^2 + m_b^2 + 2m_a m_b \cos\theta_{ab}^c$ and C_i, S_i denote numerical coupling factors and symmetry factors, respectively. All the expressions in (2.3,2.4,2.5) have in general non-meromorphic terms when a theory under consideration has a mass spectrum with more than one mass. Non-trivial cancellation of the non-meromorphic terms against one another was first observed explicitly in [30].

For $a_{2n}^{(2)}$ theories, the classical masses are

$$m_a = 2\sqrt{2}m \sin \frac{a\pi}{h}, \quad a = 1, \dots, n \quad (2.6)$$

where $h=2n+1$ is the Coxeter number for $a_{2n}^{(2)}$ and the non-vanishing three-point couplings are

$$c_{abc} = \begin{cases} \frac{\beta}{\sqrt{2h}} m_a m_b m_c, & \text{if } a+b+c = h; \\ -\frac{\beta}{\sqrt{2h}} m_a m_b m_c, & \text{if } a \pm b \pm c = 0 \end{cases} \quad (2.7)$$

The four-point couplings can be obtained via a recursion relation as follows[27, 31]:

$$c_{abcd} = \frac{\beta^2}{m^2 h} m_{ab}^2 m_{cd}^2 + \sum_f c_{abf} \frac{1}{m_f^2} c_{\bar{f}cd} \quad (2.8)$$

As a specific case, $a_6^{(2)}$ theory is considered here. This theory has three particles and their single particle reflection amplitudes at one-loop order are evaluated as follows[§]:

$$\begin{aligned}
K_1(\theta) &= 1 + \frac{i\beta^2}{4h} \left(\frac{-1sh\theta}{ch\theta - \cos\frac{0}{14}\pi} + \frac{-1/2sh\theta}{ch\theta - \cos\frac{5}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{7}{14}\pi} + \frac{-1/2sh\theta}{ch\theta - \cos\frac{9}{14}\pi} \right. \\
&\quad \left. + \frac{1sh\theta}{ch\theta - \cos\frac{12}{14}\pi} \right) + O(\beta^4) \\
K_2(\theta) &= 1 + \frac{i\beta^2}{4h} \left(\frac{-1sh\theta}{ch\theta - \cos\frac{0}{14}\pi} + \frac{-1sh\theta}{ch\theta - \cos\frac{2}{14}\pi} + \frac{-1/2sh\theta}{ch\theta - \cos\frac{3}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{7}{14}\pi} \right. \\
&\quad \left. + \frac{1sh\theta}{ch\theta - \cos\frac{10}{14}\pi} + \frac{-1/2sh\theta}{ch\theta - \cos\frac{11}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{12}{14}\pi} \right) + O(\beta^4) \\
K_3(\theta) &= 1 + \frac{i\beta^2}{4h} \left(\frac{-1sh\theta}{ch\theta - \cos\frac{0}{14}\pi} + \frac{-1/2sh\theta}{ch\theta - \cos\frac{1}{14}\pi} + \frac{-1sh\theta}{ch\theta - \cos\frac{2}{14}\pi} + \frac{-1sh\theta}{ch\theta - \cos\frac{4}{14}\pi} \right. \\
&\quad \left. + \frac{1sh\theta}{ch\theta - \cos\frac{7}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{8}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{10}{14}\pi} + \frac{1sh\theta}{ch\theta - \cos\frac{12}{14}\pi} \right. \\
&\quad \left. + \frac{-1/2sh\theta}{ch\theta - \cos\frac{13}{14}\pi} \right) + O(\beta^4)
\end{aligned} \tag{2.9}$$

where $h=7$ for $a_6^{(2)}$. Non-meromorphic terms exactly cancel out against one another.

3. Exact Boundary Reflection Matrix

For the present purpose, $a_{2n}^{(2)}$ ATFT can be best understood in terms of a Z_2 -reduction of $a_{2n}^{(1)}$ theory, where i -th simple root is identified as $(h-i)$ -th simple roots while leaving α_0 unchanged. The parent theory consists of n -particles as well as their complex conjugates. Upon the reduction, a half of the spectrum is discarded and the remaining particles become real scalars. So the S -matrices for $a_{2n}^{(2)}$ theory are crossing-symmetric and is given by[33, 34, 27]

$$S_{ab}(\theta) = \prod_{|a-b|+1}^{a+b-1} \text{step}_2 \{p\} \{h-p\} \tag{3.1}$$

where

$$(x) = \frac{sh(\theta/2 + i\pi x/2h)}{sh(\theta/2 - i\pi x/2h)}, \quad \{x\} = \frac{(x-1)(x+1)}{(x-1+2B)(x+1-2B)} \tag{3.2}$$

and the coupling dependence enters via the universal function: $B(\beta) = \beta^2 / (\beta^2 + 4\pi)$. And non-vanishing three-point couplings shown in (2.7) can be read off from the three-point

[§]Intermediate steps are omitted here. See for examples [22, 32]

couplings c_{abc} of $a_{2n}^{(1)}$ theory by mapping any of the particle indices a with $a > n$ in c_{abc} to $(h-a)$ if it exists. This implies the two models share the same matrix $J(\theta)$ defined by

$$J_a(\theta) = \sqrt{K_a(\theta)/K_{\bar{a}}(i\pi + \theta)} = K_a(\theta)/\sqrt{S_{aa}(2\theta)} \quad (3.3)$$

which is introduced in [25] so that it satisfies a simple equation for the boundary bootstrap:

$$J_c(\theta) = J_a(\theta + i\bar{\theta}_{ac}^b)J_b(\theta - i\bar{\theta}_{bc}^a) \quad (3.4)$$

θ_{ab}^c is the fusing angles and $\bar{\theta} = i\pi - \theta$. In terms of a_{2n} root systems of the parent theory, $J_i(\theta)$ for $a_{2n}^{(2)}$ theory (or $a_{2n}^{(1)}$ theory) is given by[25]

$$J_b(\theta) = \prod_{p=0}^{h-1} [2p + 1/2 + \epsilon_b]^{1/2} \sum_a (\lambda_a \cdot w^{-p} \phi_b) \quad (3.5)$$

where

$$[x] = \frac{(x - 1/2)(x + 1/2)}{(x - 1/2 + B)(x + 1/2 - B)} \quad (3.6)$$

ϵ_b is defined as follows depending on the ‘colour’ of α_b : $\epsilon_\bullet = 1$, $\epsilon_\circ = 0$. λ_a are dual vectors such that $(\lambda_a \cdot \alpha_b) = \delta_{ab}$. w is the Coxeter element and positive roots ϕ_b are specially chosen representatives of the Weyl orbits such that $w\phi_b$ are negative roots.

The single particle boundary reflection amplitude $K_i(\theta)$ can now be obtained from the defining relation (3.3). In manipulating the building blocks, the following identities are useful:

$$\{x\}_{2\theta} = [x/2]_\theta/[h-x/2]_\theta, \quad [2h+x] = [x], \quad [-x] = 1/[x] \quad (3.7)$$

For an illustration, here $a_6^{(2)}$ theory is considered. The two-particle scattering amplitudes are

$$\begin{aligned} S_{11}(\theta) &= \{1\}\{6\} \\ S_{22}(\theta) &= \{1\}\{6\}\{3\}\{4\} \\ S_{33}(\theta) &= \{1\}\{6\}\{3\}\{4\}\{5\}\{2\} \end{aligned} \quad (3.8)$$

and a little amount of work with the a_6 root space produces the single particle amplitudes $J_i(\theta)$:

$$\begin{aligned} J_1(\theta) &= [\frac{1}{2}]^{\frac{1}{2}} [\frac{3}{2}]^{\frac{3}{2}} [\frac{5}{2}]^{\frac{5}{2}} [\frac{7}{2}]^{\frac{7}{2}} [\frac{9}{2}]^{\frac{9}{2}} [\frac{11}{2}]^{\frac{11}{2}} [\frac{13}{2}]^{\frac{13}{2}} \\ J_2(\theta) &= [\frac{1}{2}]^{\frac{1}{2}} [\frac{3}{2}]^{\frac{3}{2}} [\frac{5}{2}]^{\frac{5}{2}} [\frac{7}{2}]^{\frac{7}{2}} [\frac{9}{2}]^{\frac{9}{2}} [\frac{11}{2}]^{\frac{11}{2}} [\frac{13}{2}]^{\frac{13}{2}} \\ J_3(\theta) &= [\frac{1}{2}]^{\frac{1}{2}} [\frac{3}{2}]^{\frac{3}{2}} [\frac{5}{2}]^{\frac{5}{2}} [\frac{7}{2}]^{\frac{7}{2}} [\frac{9}{2}]^{\frac{9}{2}} [\frac{11}{2}]^{\frac{11}{2}} [\frac{13}{2}]^{\frac{13}{2}} \end{aligned} \quad (3.9)$$

Inserting S and J of (3.8,3.9) into (3.3), one obtains $K_i(\theta)$, using the identities given in (3.7):

$$\begin{aligned} K_1(\theta) &= \left[\frac{1}{2}\right]\left[\frac{3}{2}\right]\left[\frac{5}{2}\right]\left[\frac{7}{2}\right]\left[\frac{9}{2}\right]\left[\frac{11}{2}\right]\sqrt{\frac{[3]}{[4]}} \\ K_2(\theta) &= \left[\frac{1}{2}\right]\left[\frac{3}{2}\right]^2\left[\frac{5}{2}\right]^2\left[\frac{7}{2}\right]^2\left[\frac{9}{2}\right]^2\left[\frac{11}{2}\right]\sqrt{\frac{[2][3]}{[5][4]}} \\ K_3(\theta) &= \left[\frac{1}{2}\right]\left[\frac{3}{2}\right]^2\left[\frac{5}{2}\right]^3\left[\frac{7}{2}\right]^3\left[\frac{9}{2}\right]^2\left[\frac{11}{2}\right]\sqrt{\frac{[1][2][3]}{[6][5][4]}} \end{aligned} \quad (3.10)$$

It is easy to see a complete agreement between the exact result given in (3.10) and the perturbative result given in (2.9) by using the following identity:

$$\frac{(x)}{(x \pm B)} = 1 \mp \frac{i\beta^2}{4h} \frac{sh\theta}{ch\theta - \cos\frac{x}{h}\pi} + O(\beta^4) \quad (3.11)$$

4. Conclusions

In this paper, for $a_{2n}^{(2)}$ affine Toda field theory defined on a half line, single particle amplitudes of the exact boundary reflection matrix corresponding to the Neumann boundary condition are constructed, hinted by the reduction idea combined with the recently introduced matrix $J(\theta)$. Specifically, for $a_6^{(2)}$ theory, the reflection amplitudes are evaluated in the formulation developed in [22] and then a hypothesised expression of $J(\theta)$ is obtained and tested against the one-loop result. One of the distinguished features of the reflection amplitudes corresponding to the Neumann boundary condition for $a_{2n}^{(2)}$ theory is the fact that they exhibit a new kind of square root singularity.

$a_{2n}^{(2)}$ affine Toda field theory is unique in the sense that despite being based on twisted algebras, their mass ratios do remain fixed under quantum corrections. This is deeply related to the fact that their root lattices are self-dual. If the square root singularities were simple poles, they would signal the existence of boundary states[8] because they are inexplicable in terms of bulk three-particle vertices, but it is believed that no boundary states are allowed to exist with the Neumann boundary condition. In practice, the existence of the square root singularity for $a_{2n}^{(2)}$ theory slows down the rate of change of the phases of single particle reflection amplitudes in the physical region (positive real line) of the complex rapidity plane, compared with those of the parent theory.

At this stage, it is not clear whether the new type of the square root singularities would appear elsewhere. It would be very interesting to find other solutions of the boundary Yang-Baxter equation, the boundary bootstrap equation and the boundary crossing-unitarity relation, which exhibit the new type of the square root singularities.

It is remarked a few striking features of integrable relativistic quantum field theory defined on a half line. Firstly, the single particle amplitudes of boundary reflection matrix at one-loop order have no ambiguity of divergences arising from such as tadpoles at all. Secondly, given a way to compute reflection amplitudes directly without the use of S -matrices as in the formulation developed in [22], one can derive scattering amplitudes via the boundary crossing-unitarity relation, not calculating them directly. In this case, n -loop single particle amplitudes of boundary reflection matrix correspond to $(n-1)$ -loop two-particle amplitudes of scattering matrix. Thirdly, boundary reflection amplitudes can have, due to three-particle vertices in the bulk, a rich structure of poles which should not be interpreted as a signal of the existence of boundary states[¶].

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[¶]A formal possibility of this picture has been discussed in [14].

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